learning techniques.[146] One of us had already suggested in a previous paper[147] that developing the capacity for invention (or "learning invention") should be understood on the background of ancient rhetorical practice and of the corresponding treatises.

This comparison with rhetorical practice, for the sake of completing our reading and explaining in more detail how indeed the progressivity of problems might lead one to invention, that is, to become capable of "inventing" positions for converting problems to equations, is the objective of another study that we are now preparing. What this article essentially provides, in this respect, is a reasonably firm basis for such a complementary study. Our purpose, indeed, is to compare in some detail the progressivity of Diophantus's treatments of problems and the necessary progressivity of rhetorical exercises; for this, we need to have a clear idea of how the problems are arranged and according to which plausible transitions. Only when this comparison will be completed shall we be in a position to re-evaluate Hankel's judgement, which really was about learning how to solve problems and not just about their factual arrangement.

Acknowledgments We thank Bernard Vitrac for his insightful remarks and corrections on a preliminary version of this article, as well as Jeffrey Oaks for having kindly checked the language and many typos in the last version.

Appendix 1: the conspectus

			STATEMENT					DERIVATIVE M.			NON DERIVATIVE METHODS										
TANNERY pg #		PB #	Nb of sought nbs	Nb of constraints	Nb of abbreviated nbs	Nb of given nbs or ratios	**Total # of positions**	Rwg method	Quasi simulation	Plassô-method	Simple method	Two-at-once method	All-together + Sum-against diff	Simulation	Simulators	Simulated constraints	in-species argument *deeper color if explicit*	Backward reasoning	END POINT OF THE CHAIN OF POSITIONS	**AMBIGUOUS** (red: serious ambiguity)
		BOOK ONE																		
16.9		1	2	2	2	2	3	2			1								eg	NO
16.24		2	2	2	2	2	3	2			1								eg	NO
18.8		3	2	2	2	2	3	2			1								eg	NO
18.26		4	2	2	2	2	3	2			1								eg	NO
20.10		5	2	2	2	4	5	4			*1*								eg	NO
22.5		6	2	2	2	4	5	4			*1*								eg	NO
24.2		7	1	1	1	3	4	3			1								ee	NO
24.21		8	1	1	1	3	4	3			1								ee	NO
26.13		9	1	1	1	3	4	3			1								ee	NO
28.6		10	1	1	1	3	4	3			1								ee	NO
30.2		11	1	1	1	3	4	3			1								ee	NO
30.22		12	4	4	4	3	5	4			1								eg	NO
32.20		13	6	6	6	4	7	6			1								eg	NO
34.25		14	2	1	2	1	4	2				2							ece	NO
36.13		15	2	2	2	4	4	3			1								ece	NO
38.2		16	3	3	3	3	5	1	3				1						ee	NO
38.19		17	4	4	4	4	6	1	4				1						ee	NO
40.10		18	3	3	3	3	6	4	1				1						ee	NO
42.2		18 alit.					5	4			1								ece	YES
42.16		19	4	4	4	4	7	5	1				1						ee	NO

[146] Indeed, arbitrary presuming that "learning" something has a straightforward meaning, which is *immediately* understandable, generally amounts to introduce an anachronism. The latter goes all the more unnoticed that it is easy to commit: we are all bent to presume that the way we learn, either by listening, writing, or reading, is universal, as if the ways by which we were taught were timeless.

[147] (Christianidis 2007, 293).

44.13		19 alit.					8	7			1							ece	YES
46.9		20	3	3	3	3													NO
46.15		20.1	2	2	2	2	3	2			1							eg	NO
46.20		20.2	2	2	2	2	3	2			1							eg	NO
46.27		21	3	3	3	4	6	5			*1*							ee	NO
46.27		21.bis	3	3	3	4	6	5			*1*							ee	NO
50.3		21 alit.	3	3	3	4	4	3			*1*							ee	NO
50.21		22	3	2	3/4	3	7	5				2						ee	NO
54.2		23	4	3	4/5	4	12	10				2						ee	NO
56.11		24	3	2	3	3	8	6				2						ece	NO
58.14		25	4	3	4	4	6	4				2						ece	NO
60.11		26	1	2	3	2	3	2			1							ece	NO
60.23		27	2	2	2	2	4	1	2				1					eg	NO
62.20		28	2	2	2	2	4	1	2				1					eg	NO
64.12		29	2	2	2	2	4	1	2				1					eg	NO
66.1		30	2	2	2	2	4	1	2				1					eg	NO
66.19		31	2	2	2	2	4	3			1							ece	NO
68.5		32	2	2	2	2	4	3			1							ece	NO
68.21		33	2	2	2	2	4	3			1							ece	NO
70.12		34	2	2	2	2	3	2			1							ece	NO
70.25		34 ii iii	2	2	2	2													NO
72.7		35	2	2	2	2	3	2			1							ece	NO
72.21		36	2	2	2	2	2	1			*1*	2?						ece	YES
74.9		37	2	2	2	2	4	3			*1*	2?						ece	YES
74.24		38	2	2	2	2	2	1			*1*	2?						ece	YES
76.11		38 i -iv	2	2	2	2													
76.26		39	1	1	1	2	14	13			1							ece/ee	NO
		44	# positions				213	147	17	0	31	10	8	0			0	0	
			# pst. der/non der				213	164			49								

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TANNERY pg #		PB #	STATEMENT: Nb of sought nbs	STATEMENT: Nb of constraints	STATEMENT: Nb of abbreviated nbs	STATEMENT: Nb of given nbs or ratios	**Total # of positions**	DERIVATIVE M.: Rwg method	DERIVATIVE M.: Quasi simulation	DERIVATIVE M.: Plassô-method	NON DERIVATIVE METHODS: Simple method	Two-at-once method	All-together + Sum-against diff	Simulation	Simulators	Simulated constraints	*in-species argument deeper color if explicit*	Backward reasoning	END POINT OF THE CHAIN OF POSITIONS	**AMBIGUOUS** (red: serious ambiguity)
		BOOK TWO																		
82		1	2	1	2	1	4	2				2							ece	NO
82.15		2	2	1	2	1	4	2				2							ece	NO
84.10		3.1	2	1	2	1	4	2				2							ece	NO
84.25		3.2	2	1	2	1	4	2											ece	NO
86.4		4	2	1	2	1	4	2				2							ece	NO
86.16		5	2	1	2	1	4	2				2							ece	NO
88.1		6	2	2	2	2	4	3			1								ece	NO
88.20		7	2	1	2	2	2					2							ece	NO
90.9		8	2	3	4	1	4	1		2	1								ee	NO
92.1		8 alit.					4	2				2							ece	NO
92.16		9	2	3	2	2	5	3				2							eg	NO
94.11		10	2	3	2	1	5	3				2							eg	NO
96.5		11	1	2	3	2	3	2			1								de	NO
98.1		11 alit					5	1		2				2	1	1			ee	NO
98.24		12	1	2	3	2	5	1		2				2	1	1			ee	NO
100.21		13	1	2	3	2	3	2			1								de	NO
102.8		13b					5	1		2				2	1	1			ee	NO
102.21		14	3	4	5	3	6	1						5	2	3			eg	NO
104.14		15	3	4	5	3	4	1						3	2	3			eg	NO
106.8		16	2	3	4	2	6	2		2				2	1	1			ee	NO

108.1	17	3	3	3	6	10*	8?				2*							ee	NO
110.6	18	3	4	3	7	12*	11?			1*								ece	NO
110.15	17 alit	3	2	3	6	9*	7*				2*							ee	NO
112.13	19	3	4	3	1	4	1		1		2							ee	NO
114.11	20	2	2	4	0	5	1		2				2	1	1			*ee*	NO
114.24	21	2	2	4	0	7	1		2				4	1	1			ee	YES
116.16	22	2	2	4	0	5	1		2				2	1	1			*ee*	NO
118.6	23	2	2	4	0	4	1		1				2	1	1			ee	NO
118.20	24	2	2	4	0	5	2						3	1/2	2	3		ee	NO
120.12	25	2	2	4	0	3	0						3	1/2	2	3		ece	NO
122.4	26	2	3	4	1	5	2						3	1	1			ee	NO
124.2	27	2	3	4	1	4	2						2	1	1			ece	NO
124.19	28.1a	2	4	4	0	3	1				2							pb	YES
124.24	28.1b	1	2	2	0	3	1		1	1								ece	NO
126.8	28.2	2	4	4	0	5			2				3	1	3	3		ece	YES
126.16	29.1a	2	4	4	0	3	1				2							pb	YES
126.21	29.1b	1	2	2	0														NO
128.13	29.2	2	4	4	0	5	1		2				2	1	2	2		ece	NO
128.13	30	2	2	4	0	4	3						1/2	1/2	2	1/2		ee	NO
130.10	31	2	3	5	0	5	3						2	2	3	2		ee	NO
132.4	32	3	3	6	0	6	1		2				3	1	2			ee	NO
132.23	33	3	3	6	0	5	1		1				3	1	2			ece	NO
134.13	34	3	3	6	0	8	1						7	1	3	4		ee	NO
136.11	35	3	3	6	0	5	1						4	1	3	3		ee	NO
	40	# positions				179	60	0	26	5	26	0	62			20	0		
		# pst. der/non der				179	86			93									

TANNERY pg #	PB #	STATEMENT: Nb of sought nbs	Nb of constraints	Nb of abbreviated nbs	Nb of given nbs or ratios	**Total # of positions**	DERIVATIVE M.: Rwg method	Quasi simulation	Plassô-method	NON DERIVATIVE METHODS: Simple method	Two-at-once method	All-together + Sum-against diff	Simulation	Simulators	Simulated constraints	in-species argument *deeper color if explicit*	Backward reasoning	END POINT OF THE CHAIN OF POSITIONS	**AMBIGUOUS** (red: serious ambiguity)
		BOOK THREE																	
138.4	1	3	3	6	0	5	1						4	2	3	4		ee	NO
140.6	2	3	3	6	0	9	2						7	1/3	3	7		ee	YES
140.21	3	3	3	6	0	6	1						5	1/3	3	5		ee	NO
142.11	4	3	3	6	0	6	1						5	1/3	3	5		ee	NO
144.4	5	3	4	7	0	8	2		1				5	2	3			ee	YES
146.1	5 aliter																		NO
146.15	6	3	4	7	0	8	2		1				5	2	3			ee	NO
148.10	6 aliter	3	4	7	0	8	3		1				3	1	2			ee	YES
150.6	7	3	3	6	0														NO
150.8	7.1	3	5	3	0	5	2		1		2							ee	NO
150.21	7.2	3	3	6	0	4	3					1						*ece*	NO
154.2	8	3	4	7	1	8	4		1				3	1/3	3			*ee*	NO
156.2	9	3	4	7	1	8	4		1				3	1/3	3			*ee*	NO
158.2	10.1	3	3	6	1	10	1						9	2	2	3		X	YES
							3						4	2	2	3		X	
158.22	10.2a	2	3	5	1														NO
158.24	10.2b	2	4	4	1														NO
160.4	10.3	3	3	6	1	6	1		2								3	*ece*	NO

160.13		11.1	3	3	6	1	10	1 3						9 4	2 2	2 2	3 3		X X	YES
162.8		11.2	2	4	4	1														NO
162.11		11.2'	1	2	2	1														NO
162.19		11.2''	1	2	2	1	3	1		2	1								ece	NO
164.7		11.3	3	3	6	1	6	1		2								3	*ece*	NO
164.19		12	3	3	6	0	7	4 2						3 5	1 1	1 1			de	YES
166.25		13	3	3	6	0	6	2						4	1	1			de	NO
168.19		14	3	3	6	0	6	1		2				3	1	2			ee	NO
170.11		15	3	3	6	0	5	2			1			2	1	1			de	NO
172.9		15al.1a	2	1	3	0	4	1		1		2							ee	NO
172.16		15al.1b	1	2	3	0	3	2			1								X	NO
174.5		15al.2a	2	2	4	0	5	1		2				2	1	1			ee	NO
174.20		15al.2b	1	2	3	0	3											3	de	NO
176.11		16.1	3	3	6	0	2					2							X	NO
176.16		16.2a	2	2	4	0	5	1		2				2	1	1			ee	NO
178.12		16.2b	1	2	3	0	6											6	de	NO
180.8		17	3	3	6	0	4	2						2	1	1			de	NO
182.4		18	3	3	6	0	4	2						2	1	1			de	NO
182.19		19	4	8	12	0	6	1						5	1	4	5		ee	NO
186		20	3	4	5	0	4	1						3	1	3			eg	NO
188.7		21	3	4	5	0	4	1						3	1	3			eg	NO
	# problems	*33*	# positions				165	45	0	16	2	6	1	80			32	15		
			# pst. der/non der				165	61			104									
	TOTAL	**117**	**# positions**				**557**	**252**	**17**	**42**	**38**	**42**	**9**	**142**			**52**	**15**		
			# pst. der/non der				**557**	**311**			**246**									

			Total # of positions	Rwg method	Quasi simulation	Plassô-method	Simple method (italics when variant 2)	Two-at-once method	All-together + Sum-against diff	Simulation			in-species argument *deeper color if explicit*	Backward reasoning	
# problems	*44*	# positions	213	147	17	0	31	10	8	0			0	0	
		# pst. der/non der	213	164			49								
# problems	*40*	# positions	179	60	0	26	5	26	0	62			20	0	
		# pst. der/non der	179	86			93								
# problems	*33*	# positions	165	45	0	16	2	6	1	80			32	15	
		# pst. der/non der	165	61			104								
TOTAL	**117**	**# positions**	**557**	**252**	**17**	**42**	**38**	**42**	**9**	**142**			**52**	**15**	
		# pst. der/non der	**557**	**311**			**246**								
				DERIV			NON DERIV								

Appendix 2: abbreviations used

The abbreviations used in this article are partly inspired by the ones used by Paul Ver Eecke in the comments accompanying his French translation of the *Arithmetica* (Diophante 1959). However, they are completed here by some specific signs for several crucial notions or procedures that are discussed for the first time in this article. Note, however, that this system of 'transcription' has intrinsic limits that become obvious in the case of operations (see below).